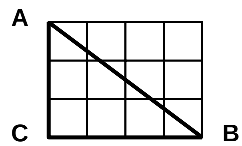
Distance between Two Points

The most common use of geographic coordinates is to measure a linear distance between two locations. For example, this can be done to calculate the length of a fence, plan a trip, or create a projected map.

Explorer's Guide

Before You Start

Using any square grid (tiled floor, ceiling or gridded paper) denote a right triangle with two perpendicular sides equal to 3 and 4 grid lengths (see figure below). Measure the length of both sides (AC and BC) and then measure the length of the third side (AB). Check if it is actually equal to 5 grid lengths. Prove it using Pythagoras theorem.



Learning by Doing

1. Outside, set up two points separated by a distance provided by instructor (e.g., 100 ft).

2. In both locations, obtain geographic coordinates (Activity 5) and convert these coordinates into decimal degrees with appropriate sign (activity 6). Record results in the following table:

Point	Longitude (Lon), ˚	Latitude (Lat), ˚
1		
2		

3. With the help of instructor, select appropriate F_{lon} and F_{lat} for your location (Activity 7):

   Average latitude = ______________ ˚
   Height above ellipsoid = ________________ m
   F_{lon} = __________________ ˚/m
   F_{lat} = ___________________ ˚/m

4. Calculate the distance using the following stepwise process:

   a. Change of longitude:
   $\Delta Lon = Lon_2 - Lon_1 =$ __________ $-$ __________ $=$ __________ $^\circ$

   b. Change of latitude:
   $\Delta Lat = Lat_2 - Lat_1 =$ __________ $-$ __________ $=$ __________ $^\circ$

   c. Distance in East-West direction:
   $\Delta X = \Delta Lon \cdot F_{lon} =$ __________ $\cdot$ __________ $=$ __________ m

   d. Distance in North-South direction:
   $\Delta Y = \Delta Lat \cdot F_{lat} =$ __________ $\cdot$ __________ $=$ __________ m

e. Distance between two points:

$$Dist = \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{\_\_\_\_\_\_\_\_\_\_^2 + \_\_\_\_\_\_\_\_\_\_^2} = \_\_\_\_\_\_\_\_\_\_\ m$$

5. Compare this calculated distance with the actual measurement. What is the difference? What percent?

$$Error = Dist_{calculated} - Dist_{measured} = \_\_\_\_\_\_\_\_\_\_ - \_\_\_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_\_\_\ m$$

$$Error_{\%} = \frac{Error}{Dist_{measured}} = \frac{\_\_\_\_\_\_\_\_\_\_}{} = \_\_\_\_\_\_\_\_\_\_\ \%$$

Example

For the point 200 ft (61 m) apart, with the following geographic coordinates:

Point	Longitude (Lon), ˚	Latitude (Lat), ˚
1	-96.468746	40.851771
2	-96.468827	40.851177

Average latitude = <u>40.851474</u> ˚ *(half of the sum of two latitudes)*
Height above ellipsoid = <u>200</u> m (can be measured with GPS)
F_{lon} = <u>84326</u> ˚/m
F_{lat} = <u>111055</u> ˚/m

$$\Delta Lon = Lon_2 - Lon_1 = \underline{-96.468827} - \underline{(-96.468746)} = \underline{-0.000081}°$$

$$\Delta Lat = Lat_2 - Lat_1 = \underline{40.851177} - \underline{40.851771} = \underline{-0.000594}°$$

$$\Delta X = \Delta Lon \cdot F_{lon} = \underline{-0.000081} \cdot \underline{84326} = \underline{-6.8}\ m$$

$$\Delta Y = \Delta Lat \cdot F_{lat} = \underline{-0.000594} \cdot \underline{111055} = \underline{-65.9}\ m$$

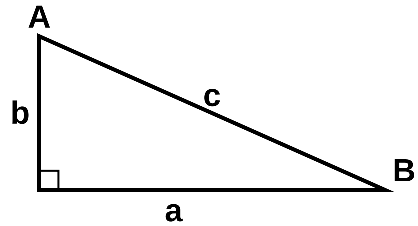
$$Dist = \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{\underline{(-6.8)}^2 + \underline{(-65.9)}^2} = \underline{66.3}\ m$$

$$Error = Dist_{calculated} - Dist_{measured} = \underline{66.3} - \underline{61.0} = \underline{5.3}\ m$$

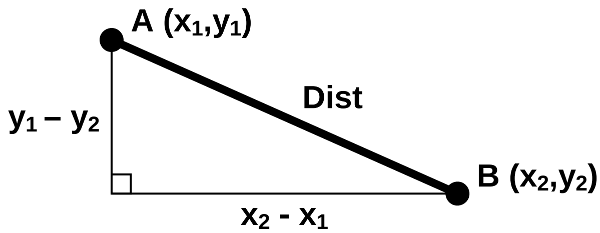
$$Error_{\%} = \frac{Error}{Dist_{measured}} = \frac{5.3}{61.0} = \underline{8.7}\ \%$$

How Does It Work

Distance between two points is a classical problem when Pythagoras theorem can be applied. According to the rule of right triangle, squared hypotenuse is equal to the sum of the squares of the other two sides, or $c^2 = a^2 + b^2$ (see figure below):



When the coordinates of A and B are given in rectangular coordinates A (x_1, y_1) and B (x_2, y_2), a and b are found as the difference between corresponding coordinates (see the figure below).



Therefore, the distance between points A and B (Dist) can be found as:

$$Dist = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

However, geographic longitude and latitude measure angles, and conversion to rectangular coordinates depend on the location (latitude and elevation). That is why the differences in geographic coordinates must be multiplied by corresponding F_{lon} and F_{lat} to find these differences in linear units:

$$x_2 - x_1 = (Lon_2 - Lon_1)\cdot F_{lon} \quad \text{and} \quad y_2 - y_1 = (Lat_2 - Lat_1)\cdot F_{lat}$$

This method works for only short distances (less than a few miles). To find distance (following the curvature of the Earth) between two points located in different parts of the world, more complex computations are needed.

Additional Challenge

For a given set of points, select one pair of geographic coordinates to represent the origin of a local XY coordinate system. Can you plot all other points with respect to this coordinate system? Make sure you use proper conversion factors and sign convention. This way, the most primitive scaled map of a given area can be created.

Interesting to Know

The length of Earth's Equator is 40,075 km (24,902 miles), which is approximately 250 times greater than the distance between Lincoln and Grand Island.

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