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Theoretical Basis for Sensor-Based In-Season Nitrogen Management

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Objective

- Sensor-based in-season management of nitrogen in corn is a popular concept
- A second-order polynomial representation of crop response to fertilization is an acceptable mathematical model
- Emerging economic constrains may affect recommended rates
- Generalized theoretical exploration of a typical cropping profitability response function may be helpful for the evolution of embedded variable rate controllers

Relative Sensor Index (RSI)

$$RSI = b_0 + b_1 \cdot VI$$

- *VI* = vegetation index (NDVI, CI, etc.)
- b_0 and b_1 = linear transformation coefficients

Sufficiently Index Example:

- b_0 = 0 and b_1 = $1/VI_{max}$
- VI_{max} = vegetation index value from a sufficiently fertilized field area

$$SI = VI / VI_{\max}$$

Crop Response Function

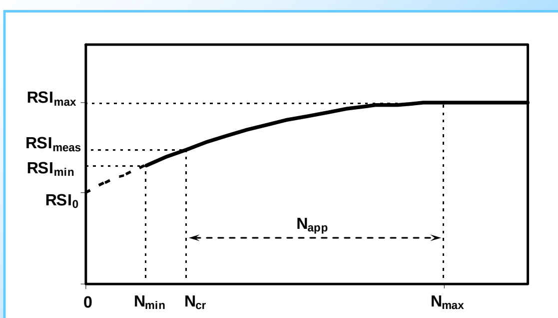
$$RSI = \begin{cases} a_0 + a_1 N + a_2 N^2 & for \quad N < N_{\max} \\ a_0 + a_1 N_{\max} + a_2 N_{\max}^2 & for \quad N \geq N_{\max} \end{cases}$$

- *N* = soil nitrogen available for crop in physical units
- c_0, c_1, and c_2 = coefficients of the second-order polynomial

Key assumptions:

- *RSI* is linearly related to crop yield for the entire range of feasible measurements (between RSI_{min} and RSI_{max})
- *N* represents the total amount of nutrient available for the crop during growing season
- Crop response does not depend on timing of fertilization

Crop Response Function



Yield Maximizing Approach

$$dRSI / dN = 0$$

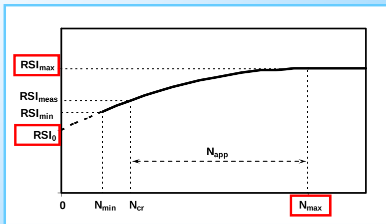
$$a_1 + 2a_2 N_{\max} = 0 \Rightarrow N_{\max} = -\frac{a_1}{2a_2}$$

- N_{max} = *N* value that corresponds to the maximum *RSI*

$$RSI_{\max} = a_0 + a_1 N_{\max} + a_2 N_{\max}^2$$

- RSI_{max} = the maximum that corresponds to VI_{max} (e.g., SI_{max} = 1)

Re-parameterisation of Second-Order Polynomial



$$a_0 = RSI_0$$

$$a_2 = \frac{RSI_0 - RSI_{\max}}{N_{\max}^2} < 0 \qquad a_1 = 2\frac{RSI_{\max} - RSI_0}{N_{\max}} > 0$$

N Availability at Sensing

$$RSI_{meas} = a_0 + a_1 N_{cr} + a_2 N_{cr}^2$$

$$N_{cr} = N_{\max} - \sqrt{N_{\max}^2 - \frac{RSI_0 - RSI_{meas}}{a_2}}$$

Maximizing RSI after Application

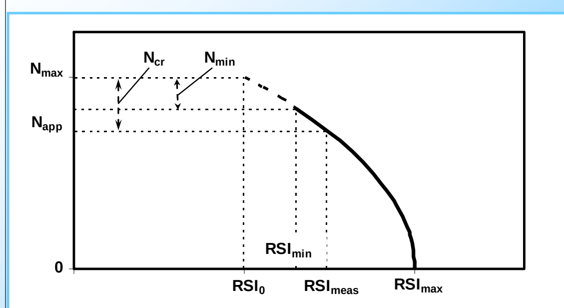
$$RSI_{res} = a_0 + a_1\left(N_{cr} + N_{app}\right) + a_2\left(N_{cr} + N_{app}\right)^2$$

$$\frac{dRSI_{res}}{dN_{app}} = a_1 + 2a_2 N_{cr} + 2a_2 N_{app} = 0$$

$$N_{app} = N_{\max}\sqrt{\frac{RSI_{\max} - RSI_{meas}}{RSI_{\max} - RSI_0}}$$

Previously derived by Holland and Schepers, 2010, Agronomy Journal 102(5):1415-1424

RSI Maximizing Function



Profit Maximizing Approach

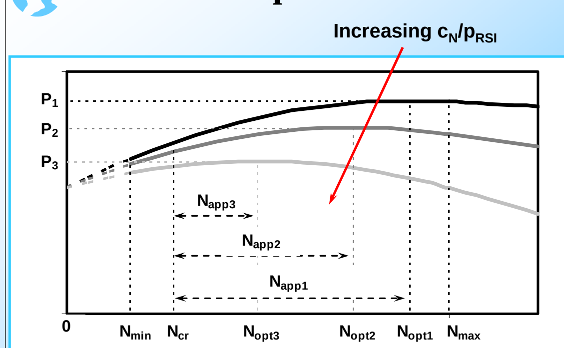
$$P = RSI_{res} \cdot p_{RSI} - N_{app} \cdot c_N$$

- ***P*** **= profit or income minus cost of fertilization, \$/ha**
- **c_N = monetary cost of N, \$/kg**
- **p_{RSI} = monetary price (value) of *RSI* associated with predicted crop yield, \$**

$$p_{RSI} = p_Y \frac{Y_{\max} - Y_{\min}}{RSI_{\max} - RSI_{\min}}$$

- **p_Y = monetary price (value) of *Yield*, \$/Mg**

Profit Response Function



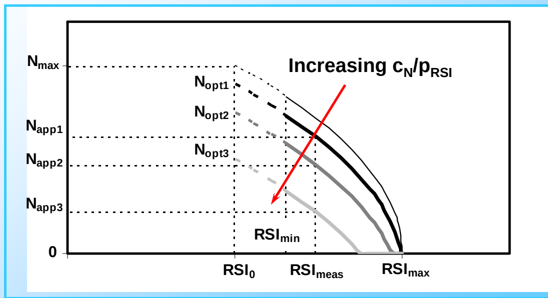
Maximizing Profit

$$P = p_{RSI}\left(a_0 + a_1\left(N_{cr} + N_{app}\right) + a_2\left(N_{cr} + N_{app}\right)^2\right) - c_N N_{app}$$

$$\frac{dP}{dN_{app}} = a_1 p_{RSI} + 2a_2 p_{RSI} N_{cr} + 2a_2 p_{RSI} N_{app} - c_N = 0$$

$$N_{app} = N_{\max}\left(\sqrt{\frac{RSI_{\max} - RSI_{meas}}{RSI_{\max} - RSI_0}} - \frac{N_{\max}}{2\left(RSI_{\max} - RSI_0\right)} \cdot \frac{c_N}{p_{RSI}}\right)$$

Profit Maximizing Function



Response	Moderate (RSI_0 = 0.8)				Strong (RSI_0 = 0.6)	
Case of c/p	Max Yield	Case 1	Case 2	Case 3	Max Yield	Case 3
N_{max}, kg/ha	*180**	*180*	*180*	*180*	*180*	*180*
RSI_0	*0.8*	*0.8*	*0.8*	*0.8*	*0.6*	*0.6*
RSI_{max}	*1*	*1*	*1*	*1*	*1*	*1*
a_0	0.8	0.8	0.8	0.8	0.6	0.6
a_1	0.0022	0.0022	0.0022	0.0022	0.0044	0.0044
a_2	-6.17E-06	-6.17E-06	-6.17E-06	-6.17E-06	-1.23E-05	-1.23E-05
N_{min}	*30*	*30*	*30*	*30*	*30*	*30*
RSI_{min}	0.86	0.86	0.86	0.86	0.72	0.72
Y_{min}, Mg/ha	12.1	12.1	12.1	12.1	10.1	10.1
Y_{max}, Mg/ha	*14*	*14*	*14*	*14*	*14*	*14*
c_N, $/kg	*0*	*1*	*2*	*3*	*0*	*3*
p_{Yield}, $/Mg	*200*	*200*	*200*	*200*	*200*	*200*
p_{RSI}, $/ha	1944	1944	1944	1944	1944	1944
c_N/p_{RSI}	0.00000	0.00051	0.00103	0.00154	0.00000	0.00154
N_{opt}, kg/ha	180	138	97	55	180	118
N_{cr}, kg/ha	*50*	*50*	*50*	*50*	*50*	*50*
RSI_{meas}	0.90	0.90	0.90	0.90	0.79	0.79
N_{app}, kg/ha	130	88	47	5	130	68

Uncertain Model Inputs

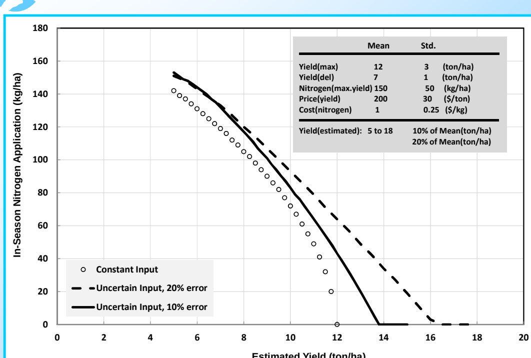
- **Crop response**
  - **Maximum yield**
  - **Yield response**
  - **Yield maximizing N**
- **Market inputs**
  - **Price of yield**
  - **Cost of fertilizer**
- **Sensor-based input**
  - **Yield predictability**

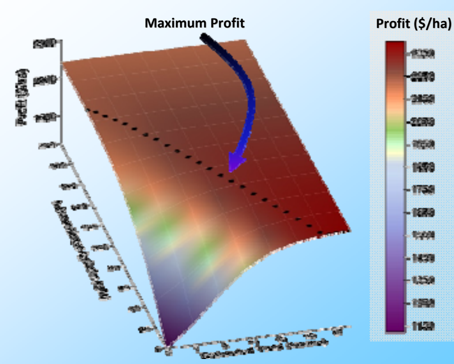
$$N_{app} = f\left(RSI_{meas}\right)$$

or

$$N_{app} = f\left(Y_{pred}\right)$$

Influence of Uncertainties





Std. of Estimated Yield: 20% of Mean

Summary

- The N recommendation algorithm is sensitive to the scale parameters of the fertilization response function
- An increase in fertilizer to sensor-based yield expectation price ratio causes a shift in the recommended application rate
- A typical deviation from a second-order polynomial response due to non recoverable yield loss of poorly-performing crop is not guided by the generic model
- Uncertainties of parameters defining the optimum application rate shift the equation to a more conservative side, which means more fertilizer in the case of nitrogen

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